PART 1: QUESTIONS

Name:________________________________ **Age:**_____ **Id:**_______________ **Course:**__________

Algebra I - Exam 3 **Lesson: 7-9**

Instructions:

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cellphone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

Exam Strategies to get the best performance:

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. Given:

I. $(b^m)^n = b^{m*n}$
II. $b^0 = 1$
III. $(ab)^m = a^m * b^m$

a) Only I is correct.
b) Only II is correct.
c) Only III is correct.
d) I, II, and III are correct.
e) None of the above.

Solution: d

I and III are exponential properties and II is an exponential definition.

2. Solve: $64^x = \frac{1}{4}$

a) $-\frac{1}{3}$
b) $-\frac{1}{4}$
c) $-\frac{1}{5}$
d) There is no solution.
e) None of the above.

Solution: a

$64^x = \frac{1}{4}$
$(4^3)^x = 4^{-1}$
$4^{3x} = 4^{-1}$
$3x = -1$
$x = -\frac{1}{3}$

3. Solve: $\sqrt{3^x} = 3^{\frac{1}{2}}$

a) 0
b) 1
c) 2
d) 3
e) 4

Solution: b

$\sqrt{3^x} = 3^{\frac{1}{2}}$
$3^{\frac{x}{2}} = 3^{\frac{1}{2}}$
$x = 1$

4. Solve: $1024^x = 0.5$

a) $-\frac{1}{2}$
b) $-\frac{1}{4}$
c) $-\frac{1}{5}$
d) $-\frac{1}{10}$
e) None of the above.

Solution: d

$1024^x = 0.5$
$(2^{10})^x = \frac{1}{2}$
$2^{10x} = 2^{-1}$
$x = -\frac{1}{10}$

5. Solve: $9^x - 10(3^x) + 9 = 0$. The solutions are:

a) $S = \{0, -2\}$
b) $S = \{0, -1\}$
c) $S = \{0, 1\}$
d) $S = \{0, 2\}$
e) None of the above.

Solution: d

$9^x - 10(3^x) + 9 = 0$
$(3^x)^2 - 10(3^x) + 9 = 0$
$y = 3^x$
$y^2 - 10y + 9 = 0 \quad (a = 1, b = -10, \text{and } c = 9)$

$S = \frac{-b}{a} \Rightarrow S = \frac{-(-10)}{1} \Rightarrow S = 10$

$P = \frac{c}{a} \Rightarrow \Rightarrow P = \frac{9}{1} \Rightarrow P = 9$

$QuickTest: \begin{matrix} +1 & +9 \\ +3 & +3 \end{matrix}$

$y = 1$ or $y = 9$

$3^x = 3^0$ $\quad 3^x = 3^2$

$x = 0$ $\quad x = 2$

Thus, $S = \{0, 2\}$

6. Solve: $3^{x-5} < 3^{7-3x}$

a) $S = \{x \in \mathbb{R} / x < 2\}$
b) $S = \{x \in \mathbb{R} / x < 3\}$
c) $S = \{x \in \mathbb{R} / x < 4\}$
d) $S = \{x \in \mathbb{R} / x < 5\}$
e) None of the above.

Solution: b

$3^{x-5} < 3^{7-3x}$
$x - 5 < 7 - 3x$
$4x < 12$
$x < 3$

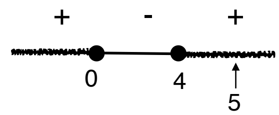
Thus, $S = \{x \in \mathbb{R} / x < 3\}$

7. Solve: $(\frac{1}{20})^{x^2} \le (\frac{1}{20})^{4x}$

a) $S = \{x \in \mathbb{R} / x \le 0 \text{ or } x \ge 2\}$
b) $S = \{x \in \mathbb{R} / x \le 0 \text{ or } x \ge 3\}$
c) $S = \{x \in \mathbb{R} / x \le 1\}$
d) $S = \{x \in \mathbb{R} / 0 \le x \le 1\}$
e) None of the above.

Solution: e

$(\frac{1}{20})^{x^2} \le (\frac{1}{20})^{4x}$
$x^2 \ge 4x$
$x^2 - 4x \ge 0$
$x(x - 4) \ge 0$



Thus, $S = \{x \in \mathbb{R} / x \le 0 \text{ or } x \ge 4\}$

8. The existence conditions for $\log_b a$ are:

a) $a < 0$ and $b < 0$.
b) $a > 0$ and $b > 0$
c) $a > 0$, $b > 0$, and $a \ne 1$.
d) $a > 0$, $b > 0$, $a \ne 1$, and $b \ne 1$.
e) $a > 0$, $b > 0$, and $b \ne 1$.

Solution: e

The existence conditions of logarithm are $a > 0$, $b > 0$, and $b \ne 1$.
For example: why is "b" different from 1?
$x = \log_1 10$
$1^x = 10$
$1 = 10$ (False)

9. Find $\log_{\frac{1}{25}} 125$ (Clue: "Double Jump")

a) -3
b) $-\frac{3}{2}$
c) $-\frac{1}{2}$
d) $-\frac{1}{3}$
e) None of the above.

Solution: b

$= \log_{\frac{1}{25}} 125 = \log_{5^{-2}} 5^3 = -\frac{3}{2}$

10. Given $\log 2 = a$ and $\log 3 = b$. Then:

I. $\log 6 = a + b$
II. $\log_3 2 = \frac{a}{b}$
III. $\log 5 = 1 + a$

Then,

a) Only I and II are correct.
b) Only I and III are correct..
c) Only III is correct.
d) Only II and III are correct.
e) I, II, and III are correct.

Solution: a

I. True.
$\log 6 = \log 2 * 3 = \log 2 + \log 3 = a + b$

II. True.
$\log_3 2 = \frac{\log 2}{\log 3} = \frac{a}{b}$ (Change of base)

III. False.
$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - a$

Thus, Only I and II are correct.

11. Solve: $\log_{\frac{1}{4}}(2 - x) = \log_{\frac{1}{4}}(x - 8)$

a) 1
b) 2
c) 3
d) There is no solution.
e) None of the above.

Solution: d

$\log_{\frac{1}{4}}(2 - x) = \log_{\frac{1}{4}}(x - 8)$; Existence $\begin{cases} 2 - x > 0 \\ x - 8 > 0 \end{cases}$
$2 - x = x - 8$
$-x - x = -8 - 2$
$-2x = -10$
$x = \frac{-10}{-2}$
$x = 5$ (Discarded)

Check: $\begin{cases} 2 - (5) < 0 \\ (-5) - 8 < 0 \end{cases}$

Thus, $S = \varnothing$.

12. Solve: $\log_2(3x - 10) = \log_2(2 - x)$.

a) There is a solution, but it is impossible to solve without a calculator or computer.
b) There is no solution.
c) 1
d) 2
e) None of the above.

Solution: b

$\log_2(3x - 10) = \log_2(2 - x)$; Existence $\begin{cases} 3x - 10 > 0 \\ 2 - x > 0 \end{cases}$
$3x - 10 = 2 - x$
$3x + x = 2 + 10$
$4x = 12$
$x = \frac{12}{4}$
$x = 3$ (Discarded)

Check: $\begin{cases} 3(3) - 10 < 0 \\ 2 - (3) < 0 \end{cases}$

Thus, $S = \varnothing$

13. Solve: $\log_3(3x - 6) \leq \log_3 2$

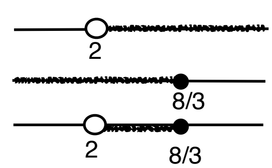
a) $S = \{x \in \mathbb{R} / 2 < x \leq 3\}$
b) $S = \{x \in \mathbb{R} / x < 2 \text{ or } x \geq 3\}$
c) $S = \{x \in \mathbb{R} / x \leq 3\}$
d) $S = \{x \in \mathbb{R} / x > 2\}$
e) None of the above.

Solution: e

$\log_3(3x - 6) \leq \log_3 2$

Existence Condition:

$3x - 6 > 0 \Rightarrow 3x > 6 \Rightarrow x > 2$



Solving we have:
$3x - 6 \leq 2 \Rightarrow 3x \leq 8 \Rightarrow x \leq \frac{8}{3}$

Thus, $S = \{x \in \mathbb{R} / 2 < x \leq \frac{8}{3}\}$.

14. Solve: $(\log_2 x)^2 - 4(\log_2 x) + 3 = 0$

a) $S = \{2\}$
b) $S = \{8\}$
c) $S = \{2 , 8\}$
d) There is no solution.
e) None of the above.

Solution: c

$(\log_2 x)^2 - 4(\log_2 x) + 3 = 0$

Existence Condition: $x > 0$

then, $y = \log_2 x$

$y^2 - 4y + 3 = 0$ $(a = 1, b = -4, \text{ and } c = 3)$

$S = \dfrac{-b}{a} \Rightarrow S = \dfrac{-(-4)}{1} \Rightarrow S = 4$

$P = \dfrac{c}{a} \Rightarrow P = \dfrac{3}{1} \Rightarrow P = 3$

$QuickTest$: $+1 \quad +3$

$y = 1$	or	$y = 3$
$\log_2 x = 1$		$\log_2 x = 3$
$\log_2 x = \log_2 2$		$\log_2 x = \log_2 8$
$x = 2$		$x = 8$

Thus, $S = \{2 , 8\}$.

15. Let $|x|$ be the absolute value of $x \in \mathbb{R}$.

I. $|x| = \begin{cases} x \text{ for } x \geq 0 \\ -x \text{ for } x < 0 \end{cases}$

II. $\left|\dfrac{xy}{z}\right| = \dfrac{|x||y|}{|z|}$

III. $|x| = (\sqrt{x})^2$; for $x \in \mathbb{R}$.

Then:

a) II and III are correct.
b) I and III are correct.
c) Only III is correct.
d) I, II, and III are correct.
e) None of the above.

Solution: e

I. True.

Absolute value definition.

II. True.

Combine the following properties.

1) $|xy| = |x||y|$

2) $\left|\dfrac{x}{y}\right| = \dfrac{|x|}{|y|}$

III. False.

$\sqrt{x} \notin \mathbb{R}$ for $x < 0$

16. Solve: $|2x - 3| = 5$

a) $S = \{-2 , -1\}$
b) $S = \{1 , 4\}$
c) $S = \{-8 , -1\}$
d) $S = \{-1 , 4\}$
e) None of the above.

Solution: d

$\|2x - 3\| = 5$	or	$2x - 3 = 5$
$2x - 3 = 5$		$2x = -5 + 3$
$2x = 5 + 3$		$2x = -2$
$2x = 8$		$x = \dfrac{-2}{2}$
$x = \dfrac{8}{2}$		$x = -1$
$x = 4$		

Thus, $S = \{-1 , 4\}$.

17. Solve: $|x+1| = |1-x|$

a) $S = \{-7, 1\}$

b) $S = \{-\frac{1}{3}, 4\}$

c) $S = \{1, 2\}$

d) $S = \{-3, \frac{3}{2}\}$

e) None of the above.

Solution: e

$|x+1| = |1-x|$

$x + 1 = 1 - x$ or $x + 1 = -1 + x$

$x + x = 1 - 1$ $\quad 1 = -1$ (Discarded)

$2x = 0$

$x = 0$

Thus, $S = \{0\}$.

18. Solve: $|x+1| = 3x - 5$

a) $S = \{\frac{1}{5}\}$

b) $S = \{-\frac{8}{5}\}$

c) $S = \{-\frac{8}{3}\}$

d) $S = \{3\}$

e) None of the above.

Solution: d

$|x+1| = 3x - 5$

Case 1: $x + 1 \geq 0$

$|x+1| = 3x - 5$

$x + 1 = 3x - 5$

$x - 3x = -5 - 1$

$-2x = -6$

$x = \frac{-6}{-2}$

$x = 3$

Check: $(3) + 1 \geq 0$ True.

Case 2: $x + 1 < 0$

$|x+1| = 3x - 5$

$-x - 1 = 3x - 5$

$-x - 3x = -5 + 1$

$-4x = -4$

$x = \frac{-4}{-4}$

$x = 1$

Check: $(1) + 1 < 0$ False.

Thus, $S = \{3\}$.

19. Solve: $|3x - 7| \leq 2$

a) $S = \{x \in \mathbb{R} / -1 \leq x \leq 4\}$

b) $S = \{x \in \mathbb{R} / x \leq -\frac{2}{3} \text{ or } x \geq \frac{10}{3}\}$

c) $S = \{x \in \mathbb{R} / x \leq -\frac{2}{5} \text{ or } \frac{8}{5}\}$

d) $S = \{x \in \mathbb{R} / \frac{5}{3} \leq x \leq 3\}$

e) None of the above.

Solution: d

$|3x - 7| \leq 2$

$-2 \leq 3x - 7 \leq 2$

$-2 + 7 \leq 3x - 7 + 7 \leq 2 + 7$

$5 \leq 3x \leq 9$

$\frac{5}{3} \leq \frac{3x}{3} \leq \frac{9}{3}$

$\frac{5}{3} \leq x \leq 3$

Thus, $S = \{x \in \mathbb{R} / \frac{5}{3} \leq x \leq 3\}$.

20. Solve: $|x-3|^2 - |x-3| = 0$

a) $S = \{1, 2, 3\}$
b) $S = \{2, 3, 4\}$
c) $S = \{3, 4, 5\}$
d) $S = \{4, 5, 6\}$
e) None of the above.

Solution: b

$|x-3|^2 - |x-3| = 0$
$y = |x-3|$
$y^2 - y = 0$
$y(y-1) = 0$
$y = 0$ or $y = 1$
$|x-3| = 0$ $\quad |x-3| = 1$
$x - 3 = 0$ $\quad x - 3 = 1$ or $x - 3 = -1$
$x = 3$ $\quad x = 1 + 3$ $\quad x = -1 + 3$
$\quad x = 4$ $\quad x = 2$

Thus, $S = \{2, 3, 4\}$.

PART 2: SOLUTIONS

Consulting

Name:______________________ **Age:**_____ **Id:**______________ **Course:**__________

Multiple-Choice Answers

Questions	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

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	Points	Max
Multiple Choice		100
Extra Points		25
Consulting		10
Age Points		25
Total Performance		160
Grade		A

Extra Questions

21. Solve: $\left(\frac{1}{3}\right)^{4x-12} \geq 3^{x^2}$

Solution: $S = \{x \in \mathbb{R} / -6 \leq x \leq 2\}$

$\left(\frac{1}{3}\right)^{4x-12} \geq 3^{x^2}$

$\left(\frac{1}{3}\right)^{4x-12} \geq \left(\frac{1}{3}\right)^{-x^2}$

$4x - 12 \leq -x^2$

$x^2 + 4x - 12 = 0 \quad (a = 1, b = 4, \text{ and } c = -12)$

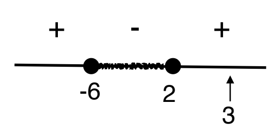
$S = \frac{-b}{a} \Rightarrow S = \frac{-4}{1} \Rightarrow S = -4$

$P = \frac{c}{a} \Rightarrow P = \frac{-12}{1} \Rightarrow P = -12$

$QuickTest:$

$+1$	-12
$+2$	-6
$+3$	-4

$x = 2$ or $x = -6$



$(x-2)(x+6) \leq 0$

Thus, $S = \{x \in \mathbb{R} / -6 \leq x \leq 2\}$

22. Solve: $\log_{\frac{1}{8}} x = \log_{\frac{1}{8}}(2x + 13)$

Solution: $S = \varnothing$

$\log_{\frac{1}{8}} x = \log_{\frac{1}{8}}(2x + 13)$

Existence Condition: $\begin{cases} x > 0 \\ 2x + 13 > 0 \end{cases}$

Then, $x = 2x + 13$

$x - 2x = 13$

$-x = 13$

$x = -13$ Discarded $\left(x > 0\right)$

Thus, $S = \varnothing$

23. Calculate P.

$P = \log_3 9 + \log_{\frac{1}{3}} 3 + \log_9 \sqrt{3}$

Solution: $P = \dfrac{5}{4}$

$P = \log_3 9 + \log_{\frac{1}{3}} 3 + \log_9 \sqrt{3}$

$P = \log_3 3^2 + \log_{(3^{-1})} 3 + \log_{(3^2)} 3^{\frac{1}{2}}$

$P = 2\log_3 3 + \dfrac{1}{-1}\log_3 3 + \dfrac{\frac{1}{2}}{2}\log_3 3$

$P = 2 - 1 + \dfrac{1}{4}$

$P = 1 + \dfrac{1}{4}$

$P = \dfrac{5}{4}$

24. Solve: $(|2x| - 16)^2 = 0$

Solution: $S = \{-8 , 8\}$

$(|2x| - 16)^2 = 0$

$y = |2x|$

$(y - 16)^2 = 0$

$y = 16$

$|2x| = 16$

$2x = 16$ or $2x = -16$

$x = \dfrac{16}{2}$ $\quad x = -\dfrac{16}{2}$

$x = 8$ $\quad x = -8$

Thus, $S = \{-8 , 8\}$

25. Find a quadratic equation by using the roots $x_1 = 2$ and $x_2 = -4$.

Solution: $x^2 + 2x - 8 = 0$

Method 1: $ax^2 + bx + c = a(x - x_1)(x - x_2)$

Then, $(x - x_1)(x - x_2) = 0$

$(x - 2)(x - (-4)) = 0$

$(x - 2)(x + 4) = 0$

$x^2 + 4x - 2x - 8 = 0$

$x^2 + 2x - 8 = 0$

Method 2: $x^2 - Sx + p = 0$ (Professor Formula)

$S = x_1 + x_2 \Rightarrow S = 2 - 4 \Rightarrow S = -2$

$P = x_1 . x_2 \Rightarrow P = (2)(-4) \Rightarrow P = -8$

Then, $x^2 - (-2)x + (-8) = 0$

$x^2 + 2x - 8 = 0$